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# A SEQUENTIAL GAME MODEL OF SPORTS CHAMPIONSHIP SERIES: THEORY AND ESTIMATION

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## Abstract

Using data from the championship series in three professional sports (basketball, baseball, and hockey), we estimate the parameters of a sequential game model for the best-of-$n$-games championship series. The unique subgame perfect equilibrium determines performance levels based on exogenous home field advantage and abilities of the two players (teams). The model provides a robust computational framework for studying strategic incentives in any sports based on identical stages (for example, single-elimination tournaments and individual tennis matches). We control for measured and unobserved differences in team strength, and we improve the small sample properties of our estimates using a bootstrap on the maximum likelihood estimates. We find negligible strategic effects in all three sports: teams in each sport play as well as possible in each game regardless of the game's importance in the series. We also estimate negligible unobserved heterogeneity after controlling for regular season records and past appearance in the championship series: teams are estimated to be exactly as strong as they appear on paper.

# I. Introduction

A sports championship series is a sequential game: two teams play a sequence of games and the winner is the team than wins more games. The sequential nature of a championship series creates a strategic element to its ultimate outcome. In this paper we solve the subgame perfect equilibrium of a sequential game model for a best-of-$n$-games championship series. In the subgame perfect equilibrium, the outcome of a series is a panel of binary responses indicating which team won which games. We estimate the parameters of the game-theoretic model using data from the championship series in professional baseball, basketball, and hockey.

The game-theoretic model nests, in a statistical sense, a model in which teams do not respond to the state of the series. In this special case, the subgame perfect equilibrium is simply a sequence of one-shot Nash equilibria, and the probability that one team wins any game depends only on home advantage and relative team ability. We formally test whether this hypothesis is supported by the data. Because each series is a short panel (at most seven games long), we apply a bootstrap procedure to the maximum likelihood estimator in an effort to reduce its small sample bias.

Our data consists of World Series since 1922, Stanley Cup finals since 1939, and NBA Championship series since 1955. We control for home field advantage and two observable measures of the teams' relative strength: the difference in the teams' regular season winning percentages and the teams' relative experience in championship series. Patterns in the data suggest that the outcomes of individual games may depend on the state of the series. In baseball, for example, 87% of World Series reaching the score three games to zero end in four games. The corresponding percentages in hockey and basketball are, respectively, 76% and 100%. These large percentages may indicate that teams that fall behind 3–0 tend to give up in the fourth game. Reaching the state 3–0 is an endogenous outcome that depends on the relative ability of the teams. Uncontrolled differences in the strengths of the teams induce positive serial correlation across the outcomes of games within a series. This serial correlation could be mistaken for dependence of outcomes on the state of the series.

However, estimates of the structural model do not support the notion that strategic incentives matter in the championship series of any of the three sports. Nor are the estimates of unobserved heterogeneity in relative team ability significant in any of the sports. The estimated strategic effect is largest in hockey, but both it and unobserved heterogeneity are still small in magnitude compared to home field advantage. In short, cliches such as a team "played with its back against the wall" or "is better than it appears on paper" are not evident in the data.

The model adapts and extends the tournament models of Lazear and Rosen (1981) and Rosen (1985) to a sequential environment. Ehrenberg and Bognanno (1990) and Craig and Hall (1994) analyze sports data in the spirit of the tournament model. Ehrenberg and Bognanno study whether performance of professional golfers is related to the prize structure of the tournament, and Craig and Hall interpret outcomes of pre-season NFL football games as a tournament among teammates for positions on their respective teams. This paper is the first application of the tournament model to sports data which imposes all of its theoretical restrictions and implications. Our theoretical results for sequential tournaments with heterogeneous competitors extend those of Rosen (1985) and Lazear (1990). In particular, by deriving the unique mixed strategy equilibrium, we can estimate a much richer model than previous theoretical work would have allowed.

## II. The Model

Let the two players (teams) in a series be called $a$ and $b$. Some aspects of the theory and empirical analysis are expressed from the perspective of the reference team, team $a$. In most elements of the model, however, team identity is arbitrary. In these cases we use the indices $t$ and $t'$ to indicate the two teams generically, $t \in \{a, b\}$ and $t' = \{a, b\} - \{t\}$. Let $j$ index the game number in the series. Our data consists of seven games series ($j = 1, 2, \ldots, 7$), but the model applies to any series length $n$, where $n$ is odd. Figure 1 illustrates the tree for a $n = 5$ playoff series. A stage of the sequential game is a game in the playoff series. An upward branch from one state indicates that team $a$ won the game and a downward branch indicates team $b$ won the game. Which branch is taken from each state is endogenous and stochastic, with the probability assigned to each branch depending on the relative performance of the teams and on pure luck (i.e. the "bounce of the ball").

The sequential game ends when one team has accumulated $(n+1)/2$ victories (in Figure 1, $(5+1)/2 = 3$). The actual length of the series is therefore endogenous and stochastic, and we denote it $n^\star$, $(n+1)/2 \leq n^\star \leq n$. Our assumptions will imply that the state of the series, denoted $\omega$, is composed of two numbers, $(n_a, n_b)$, where $n_t$ is the number of games already won by team $t$. Therefore,

$$\omega \in \Big\{ (n_a, n_b) : \quad 0 \leq \max\{n_a, n_b\} \leq (n+1)/2 \quad \& \quad 0 \leq n_a + n_b \leq n \}. \tag{1}$$

The game number can be recovered from the state since since $j = n_a + n_b + 1$.

At state $\omega$ the strategic choice variable for team $t$ is $x_{t\omega}$, interpreted as the team's performance or effort. Since each game is a one-shot simultaneous stage-game, the strategic decisions made by teams as a game progresses are not modeled. Therefore, $x_{t\omega}$ captures pre-game strategic decisions, such as which pitcher to start in baseball and any difficulties related to 'psyching up' for a game that depend on its state $\omega$.

The equilibrium choice of $x_{t\omega}$ is determined by four structural elements of the model:

$$\begin{aligned} \textit{cost of effort:} \quad & c_{tj}(x_{t\omega}) \\ \textit{score differential:} \quad & y^\star_\omega = x_{a\omega} - x_{b\omega} + \epsilon_j \\ \textit{final payoff vector:} \quad & \left( V_a[n_a, n_b] - \sum_{j=1}^{n^\star} c_{aj}(x_{a\omega}) \ , \ V_b[n_b, n_a] - \sum_{j=1}^{n^\star} c_{bj}(x_{b\omega}) \right) \end{aligned} \tag{2}$$

The cost of effort function $c_{tj}$ depends implicitly on the rules of the sport and the interaction of players, coaches, and referees. For sports as complicated as baseball, basketball, or hockey it is not possible to model the equilibrium cost of good performance as a function of the nature of the sport. For instance, if one wished to derive $c_{tj}(x_{t\omega})$ from the 'structure' of baseball, it would be necessary to model the sequential decisions made by the manager and players conditional on the score, the inning, the number of outs, the count on the hitter, the quality of the hitter relative to the pitcher and the other hitters in the batting order, etc. Instead, we exploit the common strategic elements *between* games of any best-of-$n$ series, taking as given the 'reduced-form' of the strategic elements *within* games. The cost of effort depends upon the state only through the game number $j$. For instance, $c_{tj}$ may depend upon whether $t$ is playing at home or away.[1] The final payoff for team $t$ has two components: the value the team places on the ultimate outcome, denoted $V_t[n_t, n_{t'}]$, and the total cost of effort expended during the series.

The winner of a game scores more points (or runs or goals). To determine the outcome of a series, the sign of the score difference fully determines the outcome of the game. A single game is therefore a Lazear and Rosen (1981) tournament.[2] We require only that the score index $y^\star_\omega$ in (2) be a monotonic function of the actual score difference. Linearity of $y^\star_\omega$ with respect to the effort levels is therefore less restrictive than it may appear.

The random term $\epsilon_j$ captures elements of luck in the relative performance of the two teams. The luck term is independently and identically distributed across games with distribution and density functions $F(\epsilon_j)$ and $f(\epsilon_j)$, respectively. The probabilities that team $a$ and $b$ win game $\omega$, conditional upon their chosen effort levels, can be written

$$P_{a\omega}(x_{a\omega}, x_{b\omega}) = Prob(y^\star_\omega > 0) = 1 - F(-(x_{a\omega} - x_{b\omega})) = 1 - P_{b\omega}(x_{b\omega}, x_{a\omega}). \tag{3}$$

The equilibrium level of effort also depends upon the marginal probability

$$\frac{\partial P_{a\omega}(x_{a\omega}, x_{b\omega})}{\partial x_{a\omega}} = f(-(x_{b\omega} - x_{a\omega})) = \frac{\partial P_{b\omega}(x_{a\omega}, x_{b\omega})}{\partial x_{b\omega}}. \tag{4}$$

[1] This assumption could be relaxed to allow $c$ to depend on other elements of the state of the series. For instance, the idea of "momentum" could be captured by letting $c$ depend upon the winner of the last game.

[2] In round-robin tournaments, such as the World Cup of soccer, scores within games do have a direct bearing on the ultimate champion. This means such tournaments are not tournaments in the sense introduced by Lazear and Rosen.

If the sport were a foot race with several heats, then the model has a simple interpretation (Rosen 1986). Effort $x_{t\omega}$ is the average speed of racer $t$ in heat $\omega$. Racer $t$ wins the heat if his average speed is greater than the speed of his best competitor, $t'$. The random term $\epsilon$ captures any unforeseeable events, such as cramps, that might occur during the race. A better-conditioned athlete could run any speed $x$ with less effort (lower value of $c_{t\omega}(x)$) than a worse athlete. However, the role of conditioning could not be disentangled from psychological factors having to do with competition. Hence, $c_{t\omega}$ includes the propensity for racer $t$ to 'choke' or, alternatively, to 'rise to the occasion.' In team sports, of course, effort is multi-dimensional. But in determining the ultimate outcome, effort also aggregates into a single number, the team's score.

**Assumption 1.**

[1] Cost of effort is exponential and multiplicatively separable in ability and effort:

$$c_{tj}(x_{t\omega}) = e^{-\delta_{tj}/r} e^{x_{t\omega}/r}, \tag{5}$$

for constants $\delta_{tj}$ and $r > 0$. The ability indices $\delta_{tj}$ are common knowledge.

[2] $F(\epsilon_j)$ is twice continuously differentiable and weakly quasi-concave.

[3] $F(0) = 1/2$ and $P_{t\omega}(-\infty, -\infty) = 1/2$.

[4] $\frac{1}{r} > 2f(0)$.

The negative sign in front of $\delta_{tj}$ in (5) implies that larger values of $\delta_{tj}$ are related to higher ability (lower effort costs). In the empirical specification $\delta_{tj}$ can depend upon observed and unobserved characteristics of team $t$. The sport-specific parameter $r$ determines the convexity of the cost function. As $r$ tends to zero the marginal cost of effort goes to zero. This is important special case of the model, because the winning probability (3) in the Nash equilibrium depends on just the invariant ability factors $\delta_{tj}$ and $\delta_{t'j}$. Below it is shown that the value of $f(0)$ determines effort levels in evenly matched games and that condition A1.[4] is a sufficient to rule out equilibrium in which both teams play a mixed strategy.[3]

[3] The team labels $a$ and $b$ are assigned arbitrarily, so it would be reasonable to assume

### II.2 Nash Equilibrium Effort in a Single Game

Nash Equilibrium effort of team $t$ in state $\omega$ maximizes the expected net payoff given the effort of the other team:

$$\max_{x_{t\omega}} -c_{tj}(x_{t\omega}) + E\Big[P_{t\omega}(x_{t\omega}, x_{t'\omega})\Delta V_{t\omega}\Big]. \tag{6}$$

The expectation in (6) is taken over the distribution of beliefs held by team $t$ concerning effort levels chosen by the other team, $x_{t'\omega}$. $\Delta V_{t\omega}$ is the value team $t$ places on winning the game and is determined by the Nash equilibrium in subsequent games. Three key indices associated with the state $\omega$ are

$$\begin{aligned} \textit{incentive advantage:} \quad & v_\omega \equiv \ln \frac{\Delta V_{a\omega}}{\Delta V_{b\omega}} \\ \textit{ability advantage:} \quad & \delta_j \equiv \delta_{aj} - \delta_{bj} \\ \textit{strategic advantage:} \quad & \Delta_\omega \equiv r v_\omega + \delta_j. \end{aligned} \tag{7}$$

We say that team $a$ has the *strategic advantage over team* $b$ in state $\omega$ if the index of strategic advantage is positive, $\Delta_\omega > 0$. Otherwise, team $b$ has the advantage. Strategic advantage embodies the net effect of ability advantage $\delta_j$ and incentive advantage $v_\omega$, which in turn incorporates the effect of ability advantages in future games. Proposition 1 demonstrates that $\Delta_\omega$ is indeed a proper measure of strategic advantage.

**Proposition 1.**

*[1] Under A1.[1]-A1.[3], a Nash equilibrium in mixed strategies at any stage in the series is a pair of effort levels* $(x^\star_{a\omega}, x^\star_{b\omega})$ *and mixing probabilities* $(\gamma_{a\omega}, \gamma_{a\omega})$ *such that*

$$x^\star_{t\omega} \equiv \begin{cases} r\ln\Big(f(\Delta_\omega + r\ln\frac{\gamma_{t'\omega}}{\gamma_{t\omega}})\Delta V_{t\omega} e^{\delta_{t\omega}} r\gamma_{t'\omega}\Big) & \textit{with prob. } \gamma_{t\omega} \\ -\infty & \textit{with prob. } 1-\gamma_{t\omega} \end{cases} \tag{8}$$

*for* $t \in \{a, b\}$*. Team* $t$ *plays a pure strategy (*$\gamma_{t\omega} = 1$*) if*

$$0 < \gamma_{t'\omega}\left(-rf(\Delta_\omega + r\ln\gamma_{t'\omega}) + F(\Delta_\omega + r\ln\gamma_{t'\omega})\right) - (1-\gamma_{t'\omega})/2. \tag{9}$$

that $f(\epsilon)$ is symmetric around zero, but none of the results depend upon symmetry of the luck distribution.

*Otherwise, $\gamma_{t\omega}$ solves*

$$0 = \gamma_{t'\omega}\left[rf\Big(\Delta_\omega + r\ln\frac{\gamma_{t'\omega}}{\gamma_{t\omega}}\Big) + F(\Delta_\omega + r\ln\frac{\gamma_{t'\omega}}{\gamma_{t\omega}})\right] + (1-\gamma_{t'\omega})\frac{1}{2}. \tag{10}$$

*[2] In equilibrium*

$$P_{a\omega} \equiv Prob(team\ a\ wins) = \gamma_{a\omega}\gamma_{b\omega}F\left(\Delta_\omega + r\ln\frac{\gamma_{b\omega}}{\gamma_{a\omega}}\right) + \frac{(1+\gamma_{a\omega})(1-\gamma_{b\omega})}{2}. \tag{11}$$

*[3] The Nash equilibrium effort levels (8) and winning probability (11) are unique.*

*[4] Let $t$ be the team with a strategic advantage in game $j$. Under A1.[4], team $t$ chooses greater effort than team $t'$ and follows a pure strategy ($\gamma_{t\omega} = 1$). If $|\Delta_\omega|$ is large enough then team $t'$ gives up with positive probability ($\gamma_{t'\omega} < 1$).*

**Proof:** All proofs are provided in Appendix 1.

Nash equilibrium strategies may not be pure because assumption A1.[2] assumes only quasi-concavity in the distribution of the luck factor $\epsilon$. The objective (6) may not be strictly concave: a team may prefer the boundary solution $x_{t\omega} = -\infty$ over the interior solution. If so, the other team would not prefer an interior solution either. Mixed strategies may appear to be an unnecessary complication that could be eliminated by assuming concavity in the luck distribution. However, two standard choices for the luck distribution $F$—in particular, the logistic and the normal distributions—are quasi-concave. Since the pure strategy winning probabilities are not continuous in the model's parameters when $F$ is not concave, it is important to allow for the possibility of mixed strategies.

Propositions 1.[1] also shows that exponential costs imply that $\Delta V_{t\omega}$, team $t$'s reward for winning a game, does not determine whether the equilibrium strategy is pure or mixed. The index of strategic advantage, $\Delta_\omega$, determines whether either or both teams will follow a pure strategy at state $\omega$. A cost function not exponential in effort or not separable in ability would generally not lead to such an index, which would make computation of the equilibrium less reliable. Instead, Proposition 1.[4] leads to a straightforward algorithm to compute the Nash equilibrium effort levels:

**Algorithm for Computing Nash Equilibrium**

[N1] Compute $\Delta_\omega$. If $\Delta_\omega > 0$ then team $a$ will not mix, but team $b$ may. If $\Delta_\omega < 0$ then team $b$ will not mix, but team $a$ may.

[N2] Let $t$ be the team that may mix, so $\gamma_{t'} = 1$. Check condition (9). If (9) is satisfied, then both teams follow pure strategies, i.e. they choose the interior effort levels given in (8). (Done)

[N3] If (9) is not satisfied then solve the implicit equation (10) for $\gamma_{t\omega}$. Once solved, the interior effort levels of both teams can also be computed with $\gamma_{t'} = 1$. Since the solution to (10) must lie in the range [0,1], a simple bisection method is sufficient to solve for $\gamma_{t\omega}$. (Done)

From Proposition 1.[4] we can see that whether mixed strategies are ever played in equilibrium depends on the parameter $r$ and the absolute value of ability differences $\delta_j$. We might expect that teams playing in the championship series are relatively evenly matched, since they usually are the two best teams in the league. Both incentive effects and the probability of giving up are small in a championship series compared to, say, a series between the best and worst teams. Rosen(1986) explores how the optimal structure of rewards in a promotion ladder are affected by the possibility of players giving up early on.

### II.3 Subgame Perfect Equilibrium and its Empirical Implications

To derive how strategic incentives evolve during the course of a series we must specify the value of the final outcomes. We assume that teams behave as if they only care about the ultimate winner of the series and the net costs of effort expended during the series. That is, the final payoff $V_t(n_t, n^\star - n_t)$ depends only on $\max\{n_t, n^\star - n_t\}$. (Recall that $n^\star$ is the number of games actually played). Without loss of generality we set the two final payoffs equal to $\pm 1$:[4]

[4] Teams might very well place different values on winning the series. The effect of this difference would, however, not depend upon the state of the series and would act exactly like an unobserved constant in relative ability $\delta_j$. The empirical analysis controls for unobserved differences in $\delta_j$, so setting payoffs equal is simply a normalization.

**Assumption 2.** For $(n+1)/2 \leq n^\star \leq n$,

$$V_a[\ (n+1)/2\ ,\ n^\star - (n+1)/2\ ] \quad \equiv \quad V_b[\ (n+1)/2\ ,\ n^\star - (n+1)/2\ ] \equiv 1$$
$$V_a[\ n^\star - (n+1)/2\ ,\ (n+1)/2\ ] \quad \equiv \quad V_b[\ (n+1)/2\ ,\ n^\star - (n+1)/2\ ] \equiv -1,$$

**Proposition 2.**

*[1] The unique subgame perfect equilibrium is defined as the effort functions $x^\star_{t\omega}$ in (8) and mixing probabilities $\gamma_{t\omega}$ in (9)-(10), $t \in \{a, b\}$, and*

$$\begin{aligned} V_t[n_t, n_{t'}] \equiv & \gamma_{t\omega} \Delta V_{t\omega} \Big[ -rf(\Delta_\omega + r \ln \frac{\gamma_{t\omega}}{\gamma_{t'\omega}}) + \gamma_{t'\omega} F(\Delta_\omega + r \ln \frac{\gamma_{t\omega}}{\gamma_{t'\omega}}) \Big] \\ & + \frac{(1-\gamma_{t\omega})}{2} \Big[ \gamma_{t'\omega} V_t[n_t, n_{t'}+1] + (1-\gamma_{t'\omega}) V_t[n_t+1, n_{t'}] \Big] \\ \Delta V_{t\omega} = & V_t[n_t+1, n_{t'}] - V_t[n_t, n_{t'}+1]. \end{aligned} \tag{12}$$

*[2] As $r \to 0$ the dynamics within the series disappear and the outcome of each game only depends on the ability index $\delta_j$.*

**Proof:** Backwards induction.

Proposition 2.[2] implies that the sequential game model defined by Assumptions 1 and 2 nests an intuitively appealing competing model. Namely, as $r$ goes to 0, the two teams do not respond to strategic incentives. We call this special case of the subgame perfect equilibrium *the static model*. In the static model the outcome of any game depends only upon their relative abilities (including the effect of home advantage). Teams are “professional”; they each perform as well as they can and only factors independent of the state of the series affect their relative performance. Under the static model, many common sports cliches do not apply. For instance, teams do not “play with their backs up against the wall” nor do they “taste victory.” For large values of $r$ (relative to the ability values), these cliches would apply. They may or may not apply in a given game depending upon how abilities and incentives interact to determine equilibrium effort. With Proposition 2, the notion that strategic incentives matter can be tested by simply testing whether $r$ is significantly greater than 0. The first step is to posit a specification for the cost of effort parameter $\delta_{t\omega}$.

**Assumption 3.**

$$\delta_{tj} \equiv \alpha_t + X_{tj}\beta \tag{13}$$

*where: $X_{tj}$ is a vector of observed characteristics of team $t$ in game $j$, pre-determined at the start of game 1; $\beta$ is a vector of unknown parameters that determine how strongly a team's ability is predicted by the measurable characteristics $X_{tj}$; and $\alpha_t$ is the residual ability of team $t$ not already captured by $X_{tj}$.*

In our analysis, $X_{tj}$ contains the regular season record, past appearances in the championship series (as a measure of experience), and home or away status in game $j$. Assumption 3 leads to the empirical structure for ability differences and winning probabilities:

$$\textit{observed ability advantage:} \quad X_j \equiv X_{aj} - X_{bj} \tag{14}$$

$$\textit{residual ability advantage:} \quad \alpha \equiv \alpha_a - \alpha_b \tag{15}$$

$$\textit{net ability advantage:} \quad \delta_j = \delta_{aj} - \delta_{bj} = \alpha + X_j\beta$$

$$\textit{winning probability:} \quad P_{a\omega} = \gamma_{a\omega}\gamma_{b\omega}F\Big(\alpha + \beta X_j + rv_\omega\Big) + (1+\gamma_{a\omega})(1-\gamma_{b\omega})/2. \tag{16}$$

To apply (16) to data from an observed series we must introduce notation to track the sequence of realized states. Let the variable $W_j$ take on the value 1 if team $a$ wins game $j$ of the series, and otherwise $W_j$ equals 0. Let $W = (W_1, W_2, \ldots, W_{n^\star})$ and $X = (X_1, X_2, \ldots, X_{n^\star})$ denote the sequences of outcomes and observable characteristics within a series. Then the realized state in game $j$ is

$$\omega(j) \quad \equiv \quad \left( \quad \sum_{m=1}^{j-1} W_m \quad , \quad j-1-\sum_{m-1}^{j-1} W_m \quad \right). \tag{17}$$

The probability of the observed sequence of outcomes in a single series is

$$P^\star(W, X, \alpha; \beta, r) \equiv \prod_{j=1}^{n^\star} \Big[P_{a\omega(j)}\Big]^{W_j} \Big[1 - P_{a\omega(j)}\Big]^{1-W_j}. \tag{18}$$

**Proposition 3.**

*[1]* $P^{\star}(W,X,\alpha;\beta,r)$ *is a continuous function of the estimated parameters* $\beta$ *and* $r$.

*[2] If the subgame perfect equilibrium consists of pure strategy equilibria at all states of the series, then the equilibrium generates a reduced form that is a panel data binary choice model:*

$$P_{a\omega} = Prob(y^{\star}_{\omega} > 0) = F\Big(\alpha + \beta X_j + rv_{\omega(j)}\Big). \tag{19}$$

*If* $\epsilon_j$ *is normally distributed, then the reduced form is a probit model with latent regressor* $rv_{\omega(j)}$. *If* $\epsilon_j$ *follows the logistic distribution, then the reduced form is a logit. If* $\epsilon_j$ *is uniform then the reduced form is the linear probability model.*

*[3] In the reduced form, the parameter* $r$ *is not separately identified.*

**Proof:** Immediate.

Continuity of $P^{\star}$ in the estimated parameters (3.[1]) is critical for empirical reasons, and, if attention were paid solely to pure strategies, continuity would not hold. In pure strategies, a small change in the ability index $\delta_j$ induced by a change in $r$ or an element of $\beta$ could lead to no equilibrium at all, which causes a 'jump' in the likelihood function for the data. Maximizing the likelihood function iteratively from arbitrary starting values, even if pure strategies ultimately apply, would be greatly complicated by the discontinuity.

Proposition 3.[2] makes an explicit link between the game-theoretic model and a simpler analysis of game winners using ordinary probit or logit models. That is, define the *reduced form* of the sequential game model as an analysis based on (19) in which the subgame perfect equilibrium is not solved. The reduced form is therefore a binary response model of game winners explained by the vector $X_j$ and unobserved ability difference $\alpha$. The third term of (19), $rv_{\omega}$, is a latent regressor in the reduced form. The incentive advantage $v_{\omega}$ depends implicitly on $r$, as well as $\beta$, $\alpha$ and the values of $X_k$, for $k > j$. Therefore, it is not possible to treat $rv_{\omega}$ as a typical error term (say, mean zero and heteroscedastic across the state $\omega$), because it is correlated with included variables and depends directly on other estimated parameters. Only for a special case of the sequential game model, namely the static $r = 0$ model, is the reduced form a simple probit-type model with no latent regressor. In this case the latent term disappears because both of its components go to zero. Hence, neither $r$ nor the value of $v_\omega$ can be recovered from a reduced form analysis.

In a structural analysis, the subgame perfect equilibrium is solved while estimating the parameters of the model. The incentive advantage $v_\omega$ is no longer free nor unknown, but is instead a computed value associated with each game of all series in the data. Identification of the structural model can be thought of in two steps, although it is more efficient to estimate the model in one step as our bootstrap maximum likelihood estimator does. First, calculate $v_\omega$ for all games in the data based on initial guesses for $r$, $\beta$, and the distribution of $\alpha$. Then, estimate $\beta$, $r$, and the distribution of $\alpha$ using (19) as a random effects probit. One could then iterate on these two steps until the values of the parameter estimates in the two stages agree. The parameter $r$ is identified by the model's structure if it enters (19) other than as a a multiple of $\beta$ and $\alpha$. For example, if equilibrium $v_\omega$ turned out to be proportional to $\alpha/r$ and $\beta/r$ then $r$ would not be identified, even if the subgame perfect equilibrium were solved numerically. However, $r$ does enter the indirect value of each state separately from $\alpha$ and $\beta$. (See equation (A3) in Appendix 1). Therefore, $r$ is potentially identified by outcomes through the structure of the model.

**Proposition 4.** *Let the outcomes of playoff series be generated by the sequential equilibrium. Then estimates of $\beta$ are inconsistent if the sequential equilibrium is not solved. The amount of bias increases with the cost of effort parameter $r$, holding all else constant.*

One might try to avoid Proposition 4 by approximating the incentive effect with dummy variables for the current state of the series:

$$rv_{\omega(j)} \approx \tilde{\beta} I^\star(w_j), \tag{20}$$

where $I^\star$ is a vector with elements contained in $\{-1, 0, 1\}$ that depend on the state of the series.[5] The vector $\tilde{\beta}$ would be estimated state-of-the-series effects. The problem with approximation (20) is that the strength of the incentive index $v_{\omega(j)}$ depends on the relative strength of the teams in the current and all subsequent games, $\beta X_k$, $k = j, j+1, \ldots, n$. The error in using (20) to approximate $v_{\omega(j)}$ is therefore correlated with the other regressors. Estimates of $\beta$ are still biased even with a large sample of series.[6]

# III. Analysis of Professional Sports Championship Series

### III.1 Data

The data consist of championship series in professional baseball (Major League Baseball), professional ice hockey (National Hockey League), and professional basketball (National Basketball Association). Major rule changes over the course of the last century created the modern versions of each of the sports. In each sport, we selected our sample period to include all best-of-seven series since the introduction of these rule changes. Baseball introduced the "live ball" in 1920, but the 1920 and 1921 World Series were nine-games series, so the baseball sample covers 1922–1993.[7] Professional basketball introduced the 24-second clock in the 1954-55 season, so the basketball sample covers 1955–1994. Finally, hockey introduced icing in the 1937-38 series, but the 1938 Stanley Cup was a five-game series, so the hockey sample covers 1939–1994.

The team that played at home in game 1 is coded as the reference team (team $a$ in the model section). For example, the endogenous variable $W_{jis}$ takes on the value 1 if the team that played at home in game 1 wins game $j$ of the series $i$ in sport $s$, and otherwise $W_{jis}$

[5] We estimate exactly this approximation in the next section.

[6] Interacting the indicator vector with the observable ability vector $X_j$ reduces the bias but does not guarantee that approximation error is eliminated. For example, the incentive component in one game not only depends on which team has the home advantage in this game, but also the sequence of future home advantages. Given the fixed maximum panel length of 7, including interaction terms may make the bias in estimating $\beta$ worse by including extra parameters.

[7] The 1994 World Series and 1995 Stanley Cup were not played due to strikes by the players.

equals 0. Three measures of relative team ability were also collected: an indicator for home advantage in game $j$, $\mathrm{HOME}_{ijs}$, difference in regular season records, $\mathrm{REC}_{is}$, and an indicator for differences in appearance in last year's championship series, $\mathrm{EXPER}_{is}$. The latter two variables do not vary with game number $j$. (These and other variables derived from the data are defined in Appendix 2.)

Table 1 reports summary statistics for each sport. The baseball sample includes 421 games over 72 series, the basketball sample includes 233 games over 40 series, and the hockey sample includes 299 games over 56 series. Baseball series are on average the longest: 42% of the series go to seven games, whereas 29% of the basketball series and only 17% of the hockey series go to seven games. Four-game series occur infrequently in both basketball (13% of the series) and baseball (18% of the series). By contrast, 30% of the series end in four games in hockey, the most frequent series length.

We assume the residual ability index $\alpha$ follows the normal distribution across series, $\alpha \sim N(0, \sigma^2)$, for $\sigma^2 > 0$. Under Assumption A1.[1] the value of $\alpha$ is common knowledge of the two teams. Given their information, the probability of a series of outcomes $W$ is $P^\star(W, X, \alpha; \beta, r)$, defined in (18). To the econometrician, however, the probability is

$$Q(W, X; \sigma, \beta, r) \equiv \int_{-\infty}^{\infty} P^\star(W, X, \alpha; \beta, r)\phi(\alpha/\sigma)/\sigma d\alpha \tag{21}$$

Assuming falsely that $\sigma^2 = 0$ (no unobserved heterogeneity) induces correlation between winning probabilities of different games conditional upon the observed ability factors.

In a panel data model, correlation caused by unobserved heterogeneity leads to inconsistent estimates of $\beta$. For example, we observe in the sports data that when teams are down 3-0 they usually lose the fourth game and consequently the series. This may be because teams down 3-0 give up in the situation (i.e. $v_\omega$ is large in absolute value), or because outmatched teams are more likely to reach the situation (i.e. $\alpha$ is large in absolute value), or both. The first reason is true state dependence while the second is spurious and due simply to ability differences making it likely that a series that reaches the state 3-0 has unevenly matched teams.

Our random effects estimation procedure controls for both true state dependence, created by incentive advantages, and serial correlation, created by unobserved heterogeneity. The complete specification of the structural parameters of the game-theoretic model is

$$\begin{aligned}
\delta_{ij} &= \alpha_i + \beta X_{ij} \\
&= \alpha_i + \beta_1^s \mathrm{HOME}_{ijs} + \beta_2^s \mathrm{REC}_{ai} + \beta_3^s \mathrm{EXPER}_{ai} \\
r_s &= e^{r_s^\star} \\
\sigma_s &= e^{\sigma_s^\star} \\
F(\epsilon) &= \frac{e^\epsilon}{1+e^\epsilon}.
\end{aligned} \tag{22}$$

Superscripts have been added to $\beta_k$ and subscripts have been added to $r$ and $\sigma$ to indicate that these values are estimated separately for each sport $s$. We estimate $r_s^\star$ and $\sigma_s^\star$ to avoid having a closed lower bound on the parameter space. Large negative values of $r_s^\star$ and $\sigma_s^\star$ therefore correspond to values of $r_s$ and $\sigma_s$ near 0. The luck factor follows the standard logistic distribution. All estimated values are therefore relative to the variance of random luck inherent in the sport. Based on (21) and (22), let $Q_{is}(W^{is}, X^{is}; \sigma_s, \beta^s, r_s)$ denote the predicted probability of the $i$th series in sport $s$, where superscripts have been added to the data vectors $W$ and $X$. Denote the vector of estimated parameters as $\theta$, that is the concatenation of $\beta^s$, $r_s^\star$, and $\sigma_s^\star$ for all three sports. The log likelihood function for the combined sample is

$$\mathcal{L}(\theta) \equiv \sum_s \sum_i \ln Q_{is}(W^{is}, X^{is}; \sigma_s^\star, \beta_s, r_s^\star). \tag{23}$$

Each championship series is, in effect, a short panel of observations. While maximum likelihood estimates are consistent in this context, they may not perform well in samples of the size available here.[8] One way to correct for this type of small sample problem is to perform bootstrap estimation. The sample data is randomly sampled with replacement to form ar-

[8] We conducted Monte Carlo experiments on the ML estimates of the sequential equilibrium model. Not surprisingly, we found significant bias in the ML estimates with small samples and short series. There was a strong tendency for estimates of $r_s$ to be pushed close to zero when the true values was greater than zero.

tificial data sets of the same size.[9] Let the ML estimate from the actual sample be $\hat{\theta}^{ML}$. ML estimates of $\theta$ are also obtained for each artificial data set. With the average estimated vector across re-samples denoted $\tilde{\theta}$, the parametric bootstrap estimate is defined as

$$\theta^{BS} \equiv 2\hat{\theta}^{ML} - \tilde{\theta} \tag{24}$$

### III.2 Estimates of the Sequential Game Model

Table 2 reports logit estimates of the winner of games in each sport.[10] The specifications correspond to the static $r \to 0$ model ( equivalent to $r^\star \to -\infty$). The first specification includes only the variables that enter $\delta_j$ (setting $\sigma_s = r_s = 0$ and implying no unobserved heterogeneity and no incentive effect), for each sport $s$ and maximizing $\mathcal{L}(\theta)$ over $\beta$ alone. In all three sports, the estimated coefficient on HOME is positive and significant at the 5% level. Home field advantage is largest in basketball and smallest in baseball. The estimated coefficient on the difference in regular season winning percentages (REC) is also positive in all sports, so that, other things equal, the team with the better regular season record is more likely to win than to lose any given game of a series. In baseball, however, the coefficient on REC is not significant. The estimated coefficient on relative experience in championship series (EXPER) is also positive in all three sports, but is significant only in baseball and hockey.

The second specification in Table 2 adds the normally distributed random effect $\alpha$ and frees up its standard deviation $\sigma$. The estimate of $\sigma$ implied by $\sigma^\star$ is nearly zero in baseball and hockey and is estimated very imprecisely. This suggests little evidence for unobserved heterogeneity in these sports after controlling for the observed characteristics in the teams. Only in hockey is the estimate of $\sigma$ significantly different from zero (based on a likelihood ratio test imposing $\sigma = 0$). The main effect on the other estimates is to raise slightly the estimate of home field advantage in hockey.

[9] Each series represents an observation to be sampled, not individual games within series.
[10] We also estimated the model assuming a normal distribution (with the same variance as the standard logistic). The results were nearly identical.

The third specification in Table 2 adds a set of indicator variables for the score (state) of the series, corresponding to the attempt to control for the incentive effect proposed in section II. Based on the state vector $\omega_j$ defined in (17), the variable TeamDown0-3 is defined for each game $j$ in each series as an indicator for whether the state has reached $(0,3)$ or $(3,0)$. TeamDown1-3 and TeamDown2-3 are defined similarly (see Appendix 2). All of the estimated coefficients on the state indicators are negative except for TeamDown2-3 in baseball. A negative coefficient indicates that teams on the brink of losing the series are more likely to lose, all else constant. Since unobserved heterogeneity is also controlled for, these coefficients could perhaps be picking up incentive effects. However, only in baseball are the effects significantly different from zero on their own. The coefficient estimates and $t$-ratios for the variables HOME, REC, and EXPER, however, are, for the most part, insensitive to the inclusion of score dummies. Coefficients on previous experience that were significant no longer are.

Table 3 presents various estimates of the model with the game-theoretic parameter $r_s$ estimated as well as the other parameters for each sport. These estimates require calculation of the equilibrium effort levels presented in Proposition 1 for each possible state of a series for each series in the data. The first two specifications are maximum likelihood estimates.[11] The estimate of $r$ is significantly different from zero only in hockey. In baseball and basketball the coefficient is near zero and poorly estimated. Comparing the likelihood value to that reported in Table 2 for the static model, the difference in the likelihood value when adding $r_s$ is not significant. In other words, the static model without strategic incentives is not rejected by the data. The second ML specification fixes $\sigma_s$ and $r_s$ in baseball and basketball to their values in specification 1 to determine whether their large standard errors affect the estimated standard errors of the other parameters. Precision of the other estimates within baseball and basketball are not affected by inclusion or exclusion of $\sigma$ and $r$, but standard errors in hockey are changed.

[11] Reported standard errors on the parameters are based on the outer product of the gradient matrix.

The very small maximum likelihood estimates of $r$ in each sport, implied by the large negative estimates $r^\star$ in Table 3, indicate that the incentive effects $v_j$ are not large in professional sports championship series. To explore whether this is an artifact of the series being short panels, the last column of Table 3 presents bootstrap estimates of the most general specification of the model. There are some significant differences between the ML estimates and the ML-bootstrap estimates. For instance, the value of home advantage in each sport is estimated to be greater in the bootstrap than in the ML estimates. Differences in regular season records, however, are found to be similar predictors of relative team ability. The value of past experience is slightly larger in baseball and smaller in hockey and basketball, where the effect becomes negative. The importance of unobserved heterogeneity (size of $\sigma$) is estimated to be even smaller with the bootstrap estimate. After controlling for the observed characteristics of teams, the data suggest no significant variance remaining in team abilities.

The static model with little unobserved heterogeneity provides little theoretical possibility of teams following mixed rather than pure strategies. Only if teams were greatly outmatched on paper (that is, in the observed characteristics $X_j$) would a team give up with some probability. Furthermore, they would give up in all games played away from home since the strategic advantage does not vary with the state of the series, except through home advantage. It is not surprising then that there are no instances in the data of mixed strategies at the bootstrap estimates. But mixed strategy equilibria are encountered while maximizing the likelihood function. Since we are using only the championship series in each sport, it is not unexpected that estimated differences in ability are not great enough to lead to mixed strategies in the static model. The sequential game model is easy to extend to the case of elimination tournaments — each round would be one instance of our model and different rounds would be handled as in the single elimination model of Rosen (1985). In early rounds of professional sports playoffs, mismatches are created by the design of the tournaments where the best teams start out playing the worst.

### III.3 Size of the ability and strategic effects

The bootstrap estimates of the incentive parameter $r$ are extremely small in baseball and basketball. Since the data is choosing the static model without unobserved heterogeneity for these sports, it is straightforward to measure the relative importance of the observed characteristics of the game on the probability of either team winning. For example, HOME and EXPER are both $\pm 1$ indicator variables. Since $\beta_1$ and $\beta_3$ are of similar magnitudes in these sports, past championship experience roughly cancels out the disadvantage of playing a game away from home. Furthermore, for teams with equal experience, a home advantage is equivalent to having a better regular season record of $\beta_1/\beta_3 = 16.5$ percentage points in baseball and 35.3 percentage points in baseball. One can compute the unconditional probability (at the start of game 1) of one team or the other winning the series by computing the probability of each of the branches in Figure 1.

In hockey, the bootstrap estimate of $r$ is greater than the ML estimate. Both the estimated standard error of the ML estimate and the inter-quartile range of the estimates across re-samples (reported in Table 3) indicate that the value of $r$ is not precisely estimated. To determine the relative size of $v_j$ this implies requires solving for the subgame perfect equilibrium. All aspects of the two teams and the evolution of the series determine the winning probabilities. Using the bootstrap estimates for hockey, the sequential game model was solved for each series in the hockey data. The estimated probability that team $a$ wins the first game (played at home) was computed by backwards induction. The series were then ranked in order of this initial probability. The series at the $25^{th}$, $50^{th}$, and $75^{th}$ percentiles were found. For these three series the probability of team $a$ winning in each state of the series is shown in Figure 2. The $25^{th}$ percentile is still above .60, which indicates that home ice advantage gives team $a$ an edge in game 1 even when its observable characteristics put it in the bottom quarter of the game 1 winning probabilities.

The horizontal axis in Figure 2 is the game number, yet it is almost impossible to see the difference in probabilities in games with different states. For example, game 6 can have either the state (3,2) or (2,3) so above 6 on each solid line are two points that are indistinguishable. In the series at the $75^{th}$ percentile in initial advantage, the ratio of the two probabilities of team $a$ winning is 0.9987. The ratio between either probabilities and that of game 7 (when team a regains home ice) is 1.17. The upshot is that the bootstrap estimate of $r$ in hockey, while much larger than in the other sports, is still too small to generate any significant incentive effects in the series. The effect of home advantage (as indicated by movements of the curve) and other characteristics (as indicated by the distances between the curves) swamp any strategic effects generated by the sequential nature of the playoff series.

# V. Conclusion

This paper has analyzed outcomes in professional sports championship series to explore some empirical implications of game theory. We have developed a sequential game model of best-of-$n$-games series and have estimated the model's parameters using data from three professional sports. We estimate the effect of home field advantage and differences in relative team ability revealed by differences in regular season records and previous appearances in the championship series. We use a bootstrap procedure to improve the small sample properties of the maximum likelihood estimator. We control for both unobserved differences in relative team abilities and the strategic effects on performance arising from the subgame perfect equilibrium of the sequential game. The strength of the strategic effect is determined by a single estimated parameter. We find no evidence of strategic effects in the data for any of the three sports. Only in hockey do the magnitude and imprecision of the estimate leave open the possibility of a measurable strategic effect, but the effect on winning probability is negligible when compared to, say, the effect of home field advantage. We conclude that there is no evidence that teams "give up" or get "over confident" based on the outcome of previous games in the series. We also find no evidence of unobserved heterogeneity in ability differences after controlling for regular season records and previous championship experiences. That is, teams are estimated to be just as good as they appear on paper.

Why are there no incentive effects? One possibility is that strategic interactions *within* games cancel out any incentive effects *between* games of a series. For example, team behavior may act to focus individual players on winning the current game and to ignore the larger sequential nature of the playoff series, even when winning or losing the game is nearly meaningless. Perhaps a cooperative model of teammates might explain what elements of the sport would enable this outcome to occur. Such a theoretical exercise would attempt to make our primitive parameter $r$ an endogenous function of the sport. Also, it may be that players in these series are in some sense immune to these incentives. Perhaps players who reach the highest championship in the sport do indeed play to the best of their ability regardless of the circumstances.

Two other sports applications of the model are possible. First, the model can be estimated on several rounds of single-elimination tournaments that lead to championship series, either in these sports or other sports. In earlier rounds the differences in abilities in the teams tend to be much greater. Larger difference in ability also lead to a greater likelihood of teams giving up. This suggests that any teammate interaction that mitigates strategic incentives would become less effective in earlier rounds.

Another application is to perform the same estimation procedure on tennis matches. Each game of a tennis match is similar to a championship series, except the game does not end when one player scores $(n+1)/2$ points, because a tennis game has no maximum number of points $n$. Instead, the game winner is the player that scores four or more points and leads by at least two points. Each set is, in turn, similar to a championship series, but one which relies on a cost function specified for each point rather than each game. Furthermore, strategic advantage rises and falls within a tennis match since the first point of a new game is less decisive to the ultimate outcome than the game point in the previous game. Compared to a simple championship series between teams, a tennis match between individuals may provide more leverage to identify strategic incentives.

While sports is a natural arena for testing the tournament model, the model was developed by Lazear and Rosen (1981) to study wages within firms that have workers compete for fixed-valued prizes, such as promotions or bonuses. However, there have been few direct tests of the tournament model as an explanation for wages and promotion polices within firms. The specific tournament model developed here provides a robust computational framework for studying empirically any contest between heterogeneous players composed of a sequence of identical stage games. It may therefore serve as a basis for further empirical work outside of sports.

# Appendix 1: Proof of Propositions

## Proposition 1

Proposition 1.[1] and 1.[2].

*Step 1.* Given that team $t' = b$ is choosing a mixed strategy of the form (8), the objective of team $t = a$ in choosing effort takes the form:

$$-e^{-\delta_{aj}}e^{\frac{1}{r}x_{a\omega}} + \gamma_{b\omega}F(x_{a\omega} - x_{b\omega})\Delta V_{a\omega} + (1-\gamma_{b\omega})V_a(n_a+1, n_b). \qquad (A1)$$

Under assumption A1 the objective function is strictly quasi-concave. Necessary conditions for an interior solution for teams $a$ and $b$ are therefore the first order conditions

$$e^{x_{a\omega}/r} = r\gamma_{b\omega}f(x_{a\omega} - x_{b\omega})\Delta V_{a\omega}e^{\delta_{aj}/r}$$

$$e^{x_{b\omega}/r} = r\gamma_{a\omega}f(x_{a\omega} - x_{b\omega})\Delta V_{b\omega}e^{\delta_{bj}/r}.$$

After some manipulation their ratio leads to

$$x_{a\omega} - x_{b\omega} = \Delta_\omega + r\ln\frac{\gamma_{b\omega}}{\gamma_{a\omega}}. \qquad (A2)$$

Replacing (A2) in the first order conditions leads to the interior effort levels in (8).

*Step 2.* Substituting the interior effort level (8) into (6) leads to the indirect value of the interior solution for team $a$:

$$\gamma_{b\omega}\Delta V_{a\omega}\Big(-rf(\Delta_\omega + r\ln\gamma_{b\omega}) + F(\Delta_\omega + r\ln\gamma_{b\omega})\Big) + (1-\gamma_{b\omega})V_a(n_a+1, n_b). \qquad (A3)$$

If team $a$ gives up and sets $x_{a\omega} = -\infty$ and team $b$ puts in any effort at all, then $a$ loses the game with certainty. Team $b$ puts in effort with probability $\gamma_{b\omega}$. A1.[3] handles the case in which they both give up, so the indirect value to team $a$ of giving up in game $j$ is

$$\gamma_{b\omega}V_a(n_a, n_b+1) + (1-\gamma_{b\omega})\frac{V_a(n_a+1, n_b) + V_a(n_a, n_b+1)}{2}. \qquad (A4)$$

Comparing (A3) and (A4), the interior solution is weakly preferred to giving up if

$$\Delta V_{a\omega}\left[\gamma_{b\omega}\Big(-rf(\Delta_\omega + r\ln\gamma_{b\omega}) + F(\Delta_\omega + r\ln\gamma_{b\omega})\Big) - (1-\gamma_{b\omega})\frac{1}{2}\right] \geq 0.$$

Dividing by $\Delta V_{a\omega}$ leads to the condition (9). If the inequality in (9) holds strictly, then team $a$ prefers the pure strategy and sets $\gamma_{a\omega} = 1$ in equilibrium.

*Step 3.* If (9) does not hold then team $a$ prefers the boundary solution and team $b$ would not follow the first order condition. A value of $\gamma_{a\omega}$ below 1 induces team $b$ to lower its effort level in the interior solution. At the Nash equilibrium in mixed strategies, team $a$ is indifferent between giving up and the interior solution, so (10) holds with equality.

*Step 4.* Parallel arguments can be made concerning team $b$ and the equilibrium value of $\gamma_{b\omega}$. Substituting the interior effort levels into (3) and taking into account the probabilities of giving up lead directly to (11).**QED**

Proposition 1.[3]

In a potentially more general mixed-strategy equilibrium each team chooses a distribution over all values of effort. Let $h(x_{t'\omega})$ denote the distribution chosen by team $t'$. Quasi-concavity implies that the best response for team $t$ is to put positive probability only on $-\infty$ and the unique solution to the generalized first order condition

$$e^{\frac{1}{r}x_{t\omega}} = r\int_{x_{t'\omega}} f(x_{t\omega} - x_{t'\omega})\Delta V_{t\omega} e^{\delta_{t\omega}} h(x_{t'j}) dx_{t'\omega}.$$

Given this response, unique values of $\gamma_{a\omega}$ and $\gamma_{b\omega}$ exist that satisfy (10). Therefore, the equilibrium defined in Proposition 1 is unique. **QED**

Proposition 1.[4]

Suppose the teams are equally matched ($\Delta_\omega = 0$). Then under A1.[3] $F(\Delta_\omega) = 1/2$ and effort will be symmetric. Looking at (9), equally matched teams choose pure strategies if A1.[4] holds. In an even match the sign of the luck factor $\epsilon$ determines the winner and $f(0)$ determines effort levels on the margin. As long as costs are not too convex relative to $f(0)$, evenly matched teams strictly prefer the interior solution and will not play mix strategies. Rosen (1985) recognized condition A1.[4] within a model of promotion ladders but focussed the analysis on pure strategy equilibria. If $\Delta_\omega > 0$, then $-rf(\Delta_\omega) + F(\Delta_\omega) > -rf(0) + \frac{1}{2}$, and team $a$ puts no probability on giving up. If $\Delta\omega < 0$ then team $b$ plays a pure strategy. If $|\Delta_\omega|$ is near zero then the other team is close enough not to give up completely but simply put in less effort. Only when $|\Delta_\omega|$ gets large enough will the team at a strategic disadvantage give up with positive probability. Since the team with strategic advantage sets higher interior effort and never gives up, the first part of Proposition 1.[4] follows as well.**QED**

**Proposition 4**

The issue is whether the value of the latent incentive advantage $rv_{\omega(j)}$ can be known without going through the backwards induction in (12), which in turn requires solution of the Nash equilibria in Proposition 1 for all possible states of the series. Recall that $n$ is the final game of the series. Under Assumption A2, $v_n = 0$, since both teams place a value of 2 on winning the last possible game played. The incentive advantage can be ignored *a priori* in game $n$, which might suggest using only outcomes from game $n$'s to control implicitly for the incentive advantage while estimating $\beta$. But game $n$ is played only if necessary, because the length of the series $n^\star$ is endogenous to outcomes. This creates a standard sample selection problem in restricting estimation to only game $n$'s. Correcting for the sample selection problem requires a solution to the sequential game model to compute $Prob(n^\star = n)$. Since $n^\star \geq (n+1)/2$, the sample selection problem does not occur in games 1 to $(n+1)/2$. However, the incentive advantage is only zero in these games if the cost parameter $r = 0$. Therefore, there is no game $k$ available in the data for which $v_k = 0$ *a priori* and reaching game $k$ is exogenous to the value of the unknown parameter $r$. **QED**

# Appendix 2: Definitions of Variables

$$\text{HOME}_{jis} \equiv \begin{cases} 1 & \text{if team } a \text{ is playing at home} \\ -1 & \text{if team } a \text{ is playing away.} \end{cases}$$

- $\text{REC}_{si}$ = the difference between the reference team's regular season winning percentage and its opponent's regular season winning percentage. In baseball and basketball, regular season winning percentage is defined as the number of regular season victories divided by the number of regular season games (multiplied by 100). Regular season games in hockey can end in a tie, so here winning percentage is defined as the number of regular season victories plus one-half of the number of regular season ties divided by the number of regular season games (multiplied by 100).

- $\text{EXPER}_{is}$ = 1 if the reference team played in the previous year's championship series but its opponent did not, −1 if the reference team did not play in the previous year's championship series but its opponent did, and 0 if both teams or neither team played in the previous year's championship series.

$$\text{TeamDown0-3} = \begin{cases} 1 & \text{if } \omega_j = (0,3) \\ -1 & \text{if } \omega_j = (3,0) \\ 0 & \text{otherwise.} \end{cases}$$

$$\text{TeamDown1-3} = \begin{cases} 1 & \text{if } \omega_j = (1,3) \\ -1 & \text{if } \omega_j = (3,1) \\ 0 & \text{otherwise.} \end{cases}$$

$$\text{TeamDown2-3} = \begin{cases} 1 & \text{if } \omega_j = (2,3) \\ -1 & \text{if } \omega_j = (3,2) \\ 0 & \text{otherwise.} \end{cases}$$

In basketball, all five of the series reaching the score 3–0 subsequently ended in four games. If a separate dummy variable for the score 3–0 were included in the specification of the model, the maximum likelihood estimate of the coefficient on this dummy variable would be infinity. To avoid this result, the dummy variables for 3–0 and 3–1 are combined into one variable in basketball.

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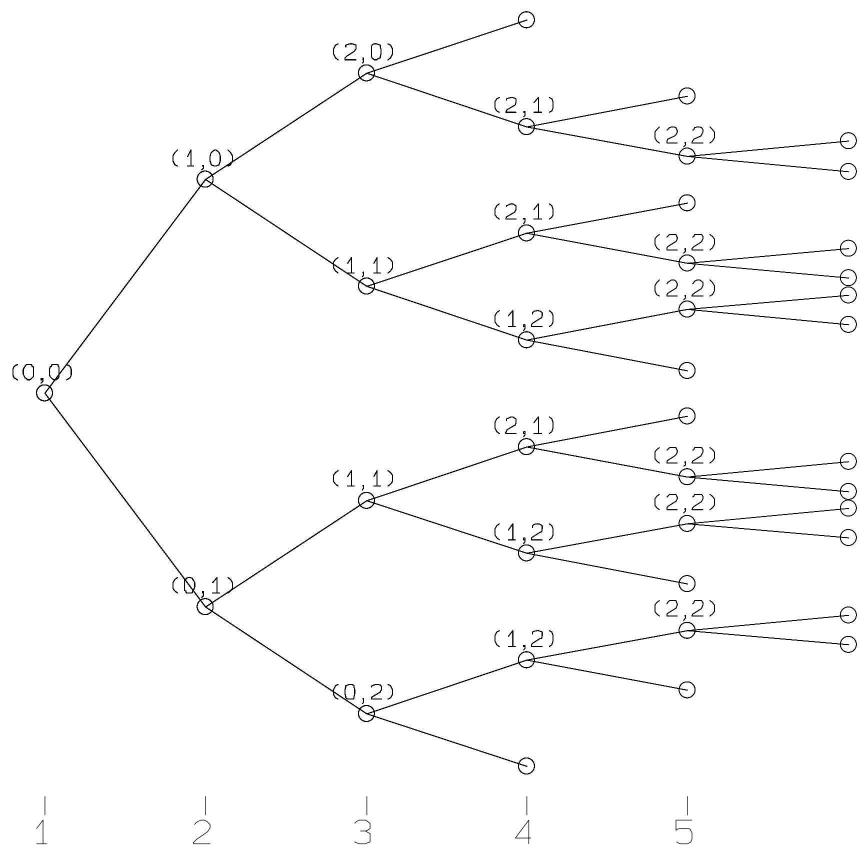

Extensive Game Form for Best-of-Five Playoff Series

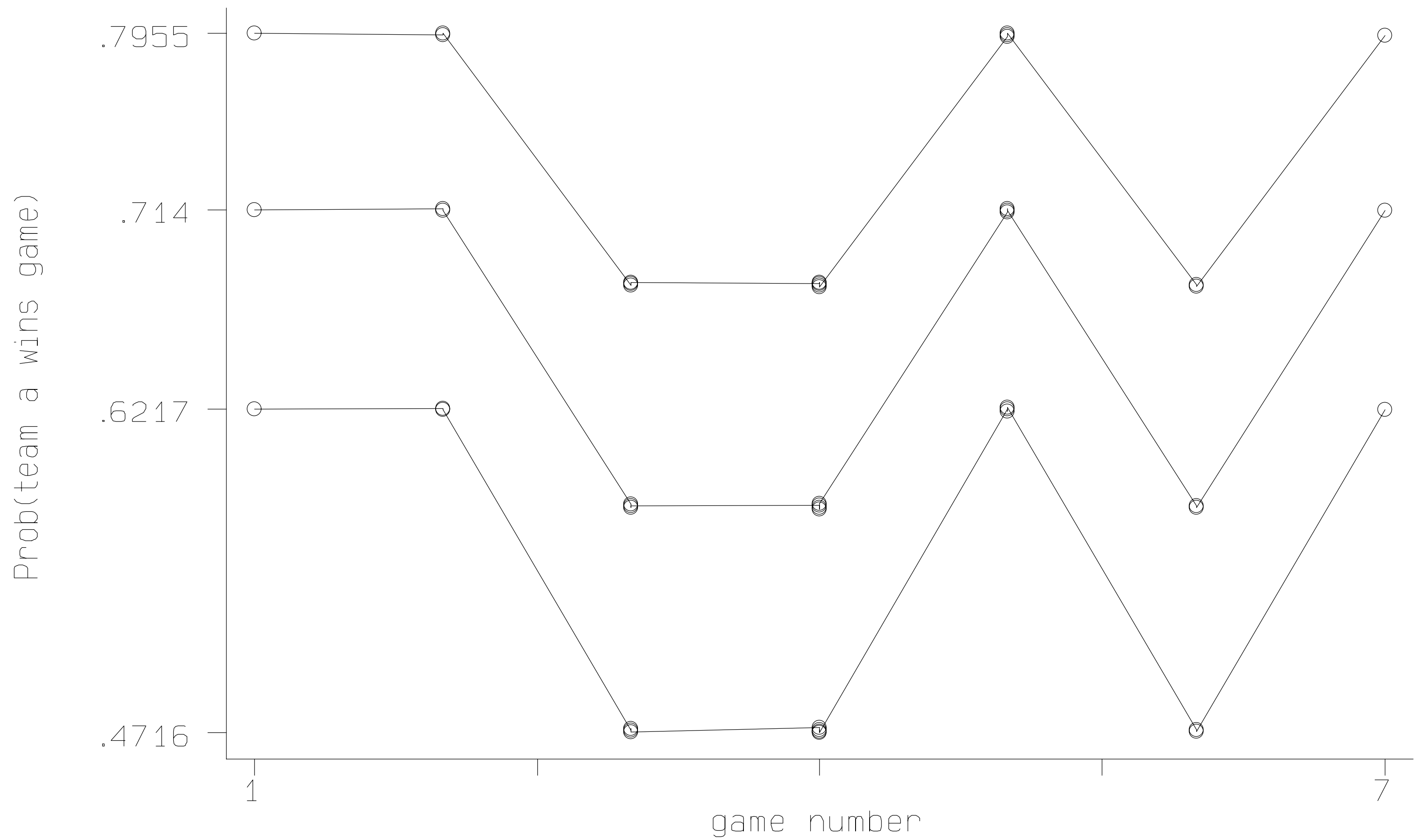


Figure2. Estimated Winning Probabilities in Hockey

Table 1. Summary of Championship Series and Games

| | Baseball World Series | Basketball NBA Finals | Hockey Stanley Cup |
|---|---|---|---|
| **Series** | | | |
| First Year | 1922 | 1955 | 1939 |
| Last Year | 1993 | 1994 | 1994 |
| Total | 72 | 40 | 56 |
| End in 4 Games | 13 | 5 | 16 |
| End in 5 Games | 15 | 9 | 15 |
| End in 6 Games | 14 | 14 | 15 |
| End in 7 Games | 30 | 12 | 10 |
| Home Sequence | HHAAAHH* | HHAAHAH** HHAAAHH*** | HHAAHAH |
| **Games** | | | |
| Total Played | 421 | 233 | 299 |
| Mean W | 0.553 | 0.588 | 0.609 |
| | (0.50) | (0.49) | (0.49) |
| Mean REC | 1.004 | 9.953 | 8.121 |
| | (4.52) | (8.17) | (8.46) |
| Mean EXPER | 0.216 | 0.172 | 0.151 |
| | (0.60) | (0.60) | (0.67) |

Sources: The Baseball Encyclopedia, Macmillan;
The Sports Encyclopedia: Pro Basketball, St. Martin's.
The National Hockey League Official Guide and Record Book, Triumph.
* other sequences were used in 1923, 1943-44, and 1961.
** = sequence until 1985, *** = sequence from 1985
Standard Deviations in ()

Table 2. Maximum Likelihood Estimates of Static Model

| | | I | | | II | | | III | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Parameter | Sport | Coeff | | Std Err | Coeff | | Std Err | Coeff | | Std Err |
| Home Field Adv. | Baseball | 0.43 | * | 0.17 | 0.43 | * | 0.17 | 0.41 | * | 0.17 |
| | Basketball | 0.66 | * | 0.26 | 0.66 | * | 0.27 | 0.69 | * | 0.26 |
| | Hockey | 0.66 | * | 0.24 | 0.72 | * | 0.22 | 0.73 | * | 0.24 |
| Record Diff. | Baseball | 0.05 | | 0.04 | 0.05 | | 0.04 | 0.04 | | 0.04 |
| | Basketball | 0.07 | * | 0.02 | 0.07 | * | 0.03 | 0.06 | * | 0.02 |
| | Hockey | 0.12 | * | 0.02 | 0.13 | * | 0.03 | 0.11 | * | 0.03 |
| Experience Diff. | Baseball | 0.54 | * | 0.28 | 0.54 | | 0.28 | 0.52 | | 0.31 |
| | Basketball | 0.19 | | 0.41 | 0.19 | | 0.62 | 0.21 | | 0.42 |
| | Hockey | 0.74 | * | 0.35 | 0.86 | * | 0.41 | 0.74 | | 0.40 |
| $\sigma$ | Baseball | | | | 0.00 | | 58.46 | 0 | | - |
| | Basketball | | | | 0.00 | | - | 0 | | - |
| | Hockey | | | | 1.19 | * | 0.47 | 0.62 | | 0.93 |
| Team Down 0-3 | Baseball | | | | | | | -3.24 | * | 1.39 |
| | Basketball | | | | | | | - | | - |
| | Hockey | | | | | | | -1.36 | | 1.16 |
| Team Down 1-3 | Baseball | | | | | | | -0.30 | | 0.69 |
| | Basketball | | | | | | | -0.90 | | 0.89 |
| | Hockey | | | | | | | -0.14 | | 0.98 |
| Team Down 2-3 | Baseball | | | | | | | 1.51 | * | 0.65 |
| | Basketball | | | | | | | -0.30 | | 0.77 |
| | Hockey | | | | | | | -1.01 | | 0.81 |
| -ln likelihood | | 606.03 | | | 605.03 | | | 595.79 | | |

Table 3. ML and ML-Bootstrap Estimates of Sequential Game Parameters

| | | ML Estimates | | | | Bootstrap ML | | |
|---|---|---|---|---|---|---|---|---|
| | | Specification 1 | | Specification 2 | | Resample | | |
| Parameter | Sport | Coeff | Std Err | Coeff | Std Err | Mean | IQ Range | Estimate |
| Home Field Adv. | Baseball | 0.43 * | 0.18 | 0.43 * | 0.17 | 0.66 | 0.34 | 0.20 |
| | Basketball | 0.66 * | 0.24 | 0.66 * | 0.27 | 1.06 | 0.44 | 0.27 |
| | Hockey | 0.72 * | 0.24 | 0.72 * | 0.22 | 0.90 | 0.51 | 0.55 |
| Record Diff. | Baseball | 0.05 | 0.04 | 0.05 | 0.04 | 0.04 | 0.07 | 0.05 |
| | Basketball | 0.07 * | 0.02 | 0.07 * | 0.03 | 0.03 | 0.02 | 0.10 |
| | Hockey | 0.13 * | 0.03 | 0.13 | 0.29 | 0.13 | 0.04 | 0.13 |
| Experience Diff. | Baseball | 0.54 | 0.29 | 0.54 * | 0.28 | 0.51 | 0.51 | 0.58 |
| | Basketball | 0.19 | 0.40 | 0.19 | 0.62 | 0.64 | 0.36 | -0.26 |
| | Hockey | 0.86 * | 0.36 | 0.86 | 1.93 | 1.01 | 0.61 | 0.71 |
| σ* | Baseball | -11.29 | -- | -11.29 | -- | -4.37 | 9.97 | -106.21 |
| | Basketball | -19.99 | -- | -19.99 | -- | -12.35 | 2.78 | -99.64 |
| | Hockey | 0.17 | 0.41 | 0.17 | 3.76 | -2.61 | 2.38 | -107.74 |
| r* | Baseball | -27.487 | 59561.0 | -27.487 | -- | -17.85 | 4.62 | -37.13 |
| | Basketball | -43.355 | 95831.8 | -43.355 | -- | -16.54 | 5.27 | -70.17 |
| | Hockey | -10.440 | 22778.0 | -10.440 | 8.8E+08 | -18.11 | 5.59 | -2.77 |
| -ln likelihood | | 605.03 | | 605.03 | | | | |

* indicates significance at the 5% level. Bootstrap estimates based on 878 re-samples of the data.
IQ Range = interquartile range of ML estimates across bootstrap resamples
Bootstrap ML Estimate = 2*ML Estimate in Spec. 1 - Resample Mean.